InnoSpaceTool Unit 9 complementary material - Noises and Losses

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In this unit we will briefly consider some important concepts when dealing with real communication systems (noisy channels). We will see what gives rise to noise physically and go over most of the terminology used to quantify noise, which we will later use in the next unit when learning how to compute the Link Budget in general and specifically for space applications.

1 Summary of Noises. Antenna Noise Temperature

We have mentioned noise several times already, but we have not looked into the causes so far. First of all, we will distinguish between coupled and non-coupled noise - the latter meaning some form of interference due to a coupling with the environment or other communication systems. Suppose that we have eliminated interference, what else could cause modification of the signals we are receiving/transmitting? Generally those are additive signals which are highly stochastic (meaning random) and arising from fundamental physical processes. They are characterized by spectral density or RMS (Root-Mean-Squared) of the voltage/current they produce. We will further distinguish between white noise and coloured noise - the former being evenly spread over the spectrum (its spectral density is a constant), the latter having some frequency-dependance. The most important noise type we will consider is caused by the motion of electrons in a resistor. At any temperature greater than absolute zero, electrons are drifting randomly, causing a non-zero voltage even if there is no external supply. The most common non-coupled noises are the following:

1. **Thermal Noise:** Known as Johnson-Nyquist noise, it is caused by random motion of charge carriers (most oftenly electrons). It is (to a very good approximation) white noise and the signal amplitude is very well modelled as a gaussian probability density function. The voltage spectral density of this noise is given by:

$$(U^2)_f = 4k_{\mathrm{B}}TR \tag{1.1}$$

Where k_{B} is Boltzman's constant ($k_{\mathrm{B}} \cong 1.38 \times 10^{-23}$[J/K]), T is the temperature in Kelvin and R is the resistance of the resistor, which causes the noise (in Ohms). For a bandwidth B (in Hz), this gives rise to a random voltage signal with RMS value of:

$$U_{RMS} = \sqrt{4k_{\mathrm{B}}TRB} \tag{1.2}$$

2. **Shot Noise:** Known as Shottky noise, it is caused by the fact that current is the result of separate charges traversing some circuit. The charges arrive at slightly different times, causing random fluctuations in the average DC current. This noise is also (to a very good approximation) white noise. Its current spectral density is given by:

$$(I^2)_f = 2q_e I \tag{1.3}$$

Where I is the DC current and q_e is the electron charge (generally that of the carrier if the shot noise is generated by other carriers as well). For a bandwidth B (in Hz), this gives rise to a random current signal with RMS value of:

$$I_{RMS} = \sqrt{2q_e IB} \tag{1.4}$$

3. **Flicker noise:** This noise is caused by resistance fluctuations, which lead to voltage and current fluctuations. Unlike the previously considered noises, the Flicker noise has a spectral density of the functional form $1/f$, which means it becomes increasingly less important at higher frequencies.

4. **Burst noise:** Known as popcorn noise, it leads to sudden and high step-like transitions between two or several discrete voltage levels (as high as a few milivolts). The cause of this noise are random trappings and releases of charge carriers.

The two leading sources of non-coupled noise in antennas are the Thermal Noise and the Shot noise, but we will only focus on the first. From 1.2 it is obvious that a resistor R in a circuit will dissipate a total power of $P = 4k_\mathrm{B}T$ per unit bandwidth (in Hz). The maximum noise power transfer happens when the rest of the circuit has its impedance matched to the resistor R. In that case each of the two resistors (R and the rest of the circuit) dissipates noise in both itself and the other resistor. Since only half of the voltage drops along any of these resistors, the resulting noise power per unit bandwidth is:

$$P = k_\mathrm{B}T \tag{1.5}$$

This is an important result, which we will use when evaluating the Carrier-to-Noise ratio in a link!
The most common coupled noises are the following:

1. **Interference:** A modification of the signal due to interference with other signal(s) along the channel.

2. **Crosstalk:** An undesired interference between two seperate channels or circuits in a singe communication system.

3. **Atmospheric noise:** A coupling between the communication system and the atmosphere, which causes random signals generated by lightning and other electrical disturbances to interfere with signals in the system.

4. **Intermodulation noise:** Caused by signals of different frequencies sharing a non-linear medium.

5. **Extraterrestrial noise:** Caused by signals received from outside of Earth's atmosphere - Solar activity, Cosmic radiation, etc.

Note that all of them can be characterized by some coupling constant of the antenna with the corresponding noise source. Their effects are not as easy to evaluate as the thermal noise and for that reason a much more concise and useful noise evaluation scheme is adopted, that of the antenna noise temperature:
Def: *The Antenna Noise Temperature (T_A) is the temperature of a hypothetical resistor at a noise-free channel which would generate the same noise power output per unit bandwidth as that generated by the real antenna.*
This effective temperature is not the physical temperature of the antenna, but it allows us to easily combine all the noises generated by the antenna coupling to its environment in a single parameter, which is much easier to work with! In terms of the Antenna Noise Temperature, the noise power (in Watts) is given by:

$$N = k_{\mathrm{B}} T_A B \tag{1.6}$$

This concept can easily be extended to include all the noises in a link, and the noise temperature corresponding to that will be used in the next unit - the system temperature T_s, which has the same interpretation as T_A but for the whole system!

2 Shannon - Hartley theorem

In the previous units we considered the bit rate of a communication link, based on the type of modulation, carrier frequency and signal bandwidth, but in all these cases we assumed zero noise power. This is not true and any practical application needs to consider the noises involved. One of the most important quantities that qualifies the noise in a channel is the Signal-to-Noise ratio (or Carrier-to-Noise ratio for the digital modulations we are considering). Depending on the situation, we will denote this dimensionless quantity by S/N or C/N respectively. This quantity is important for the following theorem.
Theorem: *Given a channel with a bandwidth B, subject to additive gaussian white noise with signal-to-noise ratio S/N, the highest possible information transmission with an arbitrarily low error rate is given by:*

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \tag{2.1}$$

This is known as the Shannon - Hartley theorem and it is a special case of the noisy-channel coding theorem, appied to an analog channel subject to Gaussian noise. C is the channel capacity in this expression (we will sometimes be using C for the carrier power also, but it is obvious which one the letter represents from the contex of the formula). We will use this theorem to evaluate the errors present in our link and to define several important quantities for it.
Note: Shannon's theorem gives us the highest possible information transmission rate, but it does not tell us how to achieve it. Implementation should still be taken into consideration for a real system!

Shannon's theorem can be rewritten in a way which introduces two new important concepts - E_b (energy per bit) and N_0 (noise power spectral density). Since the noise is white, it can be expressed in terms of its power spectral density N_0 as $N = N_0 B$, where B is once again the bandidth. Similarly, the total carrier power for digital modulation can be expressed as the transmission rate R, multiplied by the energy per bit $S = E_b R$. Then 2.1 can be rewritten as:

$$\frac{C}{B} = \log_2 \left(1 + \frac{E_b R}{N_0 B} \right) \tag{2.2}$$

The new quantity E_b/N_0 is very important since it is directly related to the BER (Bit Error Rate) of a link! It is called energy per bit over noise power spectral density (most commonly just reffere to as E_b/N_0) and it can be easily expressed as a function of the signal-to-noise ratio:

$$\frac{E_b}{N_0} = \left(\frac{S}{N} \right) \frac{B}{R} \tag{2.3}$$

The figure bellow gives some dependencies between the Bit Error Rate and E_b/N_0 for different modulations. The relation between the two is given by the complementary error function (erfc) with a factor, depending on the modulation scheme:

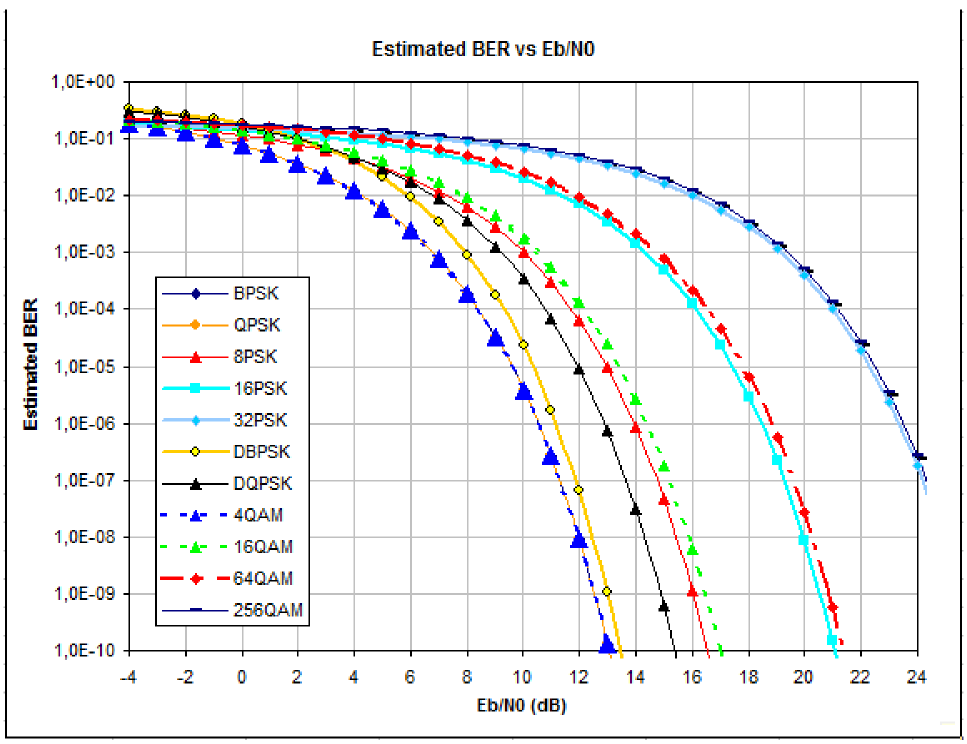

Fig.9.1 - Bit Error Rate as a function of E_b/N_0 in dBs for different modulation schemes.

The general dependancy is obvious - the larger the energy per bit per noise power spectral density, the lower the bit error rate! If there was no noise at all (corresponding to $E_b/N_0 \to \infty$), there would be no bit errors! Notice that the ratio is given in dBs in the graph, which means that each 3dBs correspond to E_b/N_0 being twice as high, remembering the second

part of unit 6.
Equation 2.2 can be used to evaluate the minimal required E_b/N_0 for any system that transfers information reliably, by considering the gross bit rate to be equal to the net bit rate $R = C$. Considering this case and taking the limit $B \to \infty$, which is known as Shannon's limit, 2.2 gives us the value:

$$\left.\frac{E_b}{N_0}\right|_{B=\infty} = \lim_{B\to\infty} \frac{2^{\frac{C}{B}} - 1}{\frac{C}{B}} = \ln 2 \cong -1.59\text{dB} \tag{2.4}$$

Note: This is the theoretical minimum limit for an infinite bandwidth ($B \to \infty$). The real Shannon limit for a finite bandwidth is always greater than this!

3 Free Space Losses

In the next unit we will consider all the losses and gains in a communication system and learn how to determine E_b/N_0 for it. Knowing this, we will be able to determine the Bit Error Rate that system would experience and perhaps modify it so that it is not too high for a meaningful communication (depending on the modulation scheme and error-correcting code)! Before that, however, we need to consider the highest losses for space applications - those of the wave propagation in free space. We already saw that the power density of antennas drop the further we go from them, but let's demonstrate it with the following simple example: consider a perfect isotropic antenna - one that emits spherical wavefronts with no preferred direction. Given a total power P_r for such an isotropic radiator, the power density is (as the name suggests), P_r divided by the surface it is spread on. Since the pattern of such an ideal radiator is a perfect sphere, the power density at a distance r is given by:

$$P_d = \frac{P_r}{4\pi r^2} \tag{3.1}$$

As expected, it is decreasing as the square of the radial distance from the source (similarly to the dipole antenna we considered back in unit 5. Now let us consider a real antenna with some directivity greater than one. We cannot directly apply this formula to its power (since its power density is a function of direction), but we can consider an Equivalent Isotropic Radiator (EIR), which as its name suggests, is an isotropic antenna which would give us the same power in any direction as the real antenna we're considering in the direction we're interested. Using our definition of directivity from unit 6, The EIR power in the direction of interest (EIRP) is simply the transmitting antenna directivity D_t, multiplied by the transmitted power P_t:

$$\text{EIRP} = D_t P_t$$

This is only true if the antenna has no radiation losses (in which case its directivity is proportional to its area). Generally, the parameter used to characterize the EIRP is the transmitting antenna gain G_t, which is simpy its directivity times the radiation efficiency: $G_t = D_t \eta_{rad}$. Then the real formula for the EIRP is:

$$\text{EIRP} = G_t P_t \tag{3.2}$$

Considering the mathematical replacement of our antenna with its equivalent isotropic antenna, we can get an expression for the received power density P_d a distance r from the source:

$$P_d(r) = \frac{G_t P_t}{4\pi r^2} \tag{3.3}$$

The r in the denominator shows us that this power is decreasing quadratically as expected, and this decrease can be interpreted as loss due to propagation. We have assumed that the propagation medium itself is vacuum in this expression, otherwise there would be additional losses!

4 Additional Losses

Additionl losses should be considered in any practical applications and the main contribution of those (especially for space applications) come from the atmosphere. Rain drops, ice and different ionospheric activity can cause signals to lose a lot more power than what would be expected simply due to their propagation. The atmosphere is a relatively thin layer around the Earth (about 50-200 km), but depending on the satellite's height above the horizon, radio waves may be passing through a much thicker effective atmosphere. Unlike the free space power reduction, which only depends on the distance, atmospheric losses depend highly on the considered frequency and have a more statistical character, since different weather conditions can change them dramatically. Generally the atmospheric attenuation is caused by droplets absorbing or changing the polarization of radio waves passing through them. This effect is usually given in dB/km (dB per kilometer of atmosphere traversed) and it is generally higher for higher-frequency bands. The decibels should not scare you - as mentioned before many quantities in radio-electronics are preferred in dB due to the otherwise high dynamic range. We will consider all of the equations derived in this unit in decibel notation when we calculate the Link Budget. Going between dB and regular units is the same for any quantity:

$$F_{\text{dBs}} = 10 \log_{10} F \tag{4.1}$$

The figure bellow shows typical values for the losses in horizontal polarization antenna (again in dB/km) for different rain strength (usually measured in mm/cm^2) and different bands.

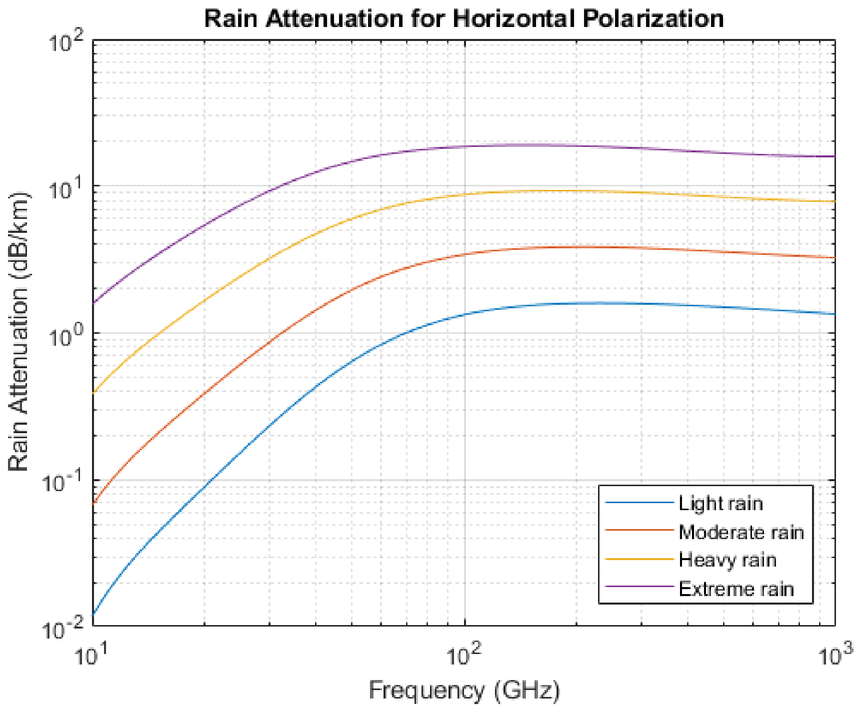

Fig.9.2 - Rain attenuation for different bands and rain strength (logarithmic scale) (replace)

We can see that this rain attenuation reaches a maximum around 100 GHz (W band), but the difference in losses for the frequencies we have been considering could be significant. More details on the attenuation caused by different weather conditions and in general absorption in the atmosphere can be found on the official website of ITU - for example ITU-R P.838-3. Typically different geographical areas are characterized by a rain distribution, which can tell us at roughly what percent of the time we can expect rain strength above or bellow some reference. This means that in practice we are always considering statistical probability of having enough power and depending on the application and geographical location, we choose a different atmospheric loss as a reference for the worst case scenario.
Two more losses which should be considered are polarization mismatch and pointing inaccuracies. In the third part of unit 3 we saw that electromagnetic waves are generally elliptically polarized, but we can use linear or circular polarization to avoid interference between emission and receiving. Suppose now that we are emitting with horizontally polarized waves that somehow got depolarized on the way to the receiver. Since the receiver will only be capturing the projection of our now elliptically polarized signal along the horizontal polarization, at any given moment it will receive electric or magnetic field which is the projection of the actual direction by the horizontal polarization direction, given by the cosine of the angle between them (denoted by α) multiplied by the field's magnitude:

$$E_r = E_0 \cos\alpha$$

Since the power of the EMW is proportional to the square of the electric (or magnetic) field, the total power received will be reduced by a factor of $\cos^2\alpha$:

$$P_r = P_0 \cos^2\alpha \tag{4.2}$$

This is clearly a reduction for any non-zero α, since $\cos^2 \alpha \in [0, 1]$. If we assume our signal got depolarized randomly (it is now elliptically polarized), then the angle should be averaged over a full period to obtain the factor, giving us 1/2:

$$P_r = \frac{1}{2} P_0 \tag{4.3}$$

The same result is valid for circularly polarized light, and so the polarization mismatch loss is generally taken to be 1/2 or -3 dB as we showed back in unit 5.
Since our formula for the Equivalent Isotropic Radiator Power 3.2 takes as its input the directivity (which is the maximum value of the directive gain, recalling what unit 6) and our antenna might not be perfectly pointed, additional losses of 1-2 dB are usually taken to make sure that the worst case scenario is taken into consideration. The exact value of those losses of course depend on the directive gain function of the antenna and the pointing accuracy itself, but the values 1-2 dB can be taken as a rule of thumb for our purposes.